

Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Monday 18 October 2021 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours

plus your additional time allowance

YOU MUST HAVE:

the Printed Answer Booklet

the Insert

a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

Write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

Fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 75.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

BLANK PAGE

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$(n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$(|x| < 1, n \in \mathbb{R})$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

$$\text{Trapezium rule: } \int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\},$$

$$\text{where } h = \frac{b - a}{n}$$

The Newton-Raphson iteration for solving

$$f(x) = 0: x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$\text{OR } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx}$$

$$\text{where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

$$\text{Standard deviation, } s = \sqrt{\text{variance}}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

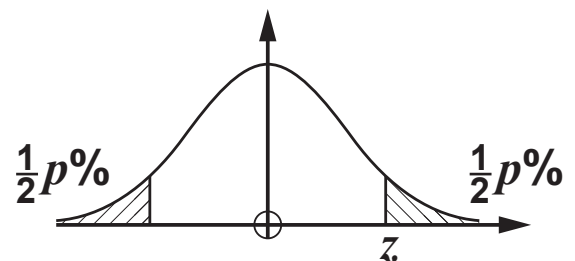
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer ALL the questions.

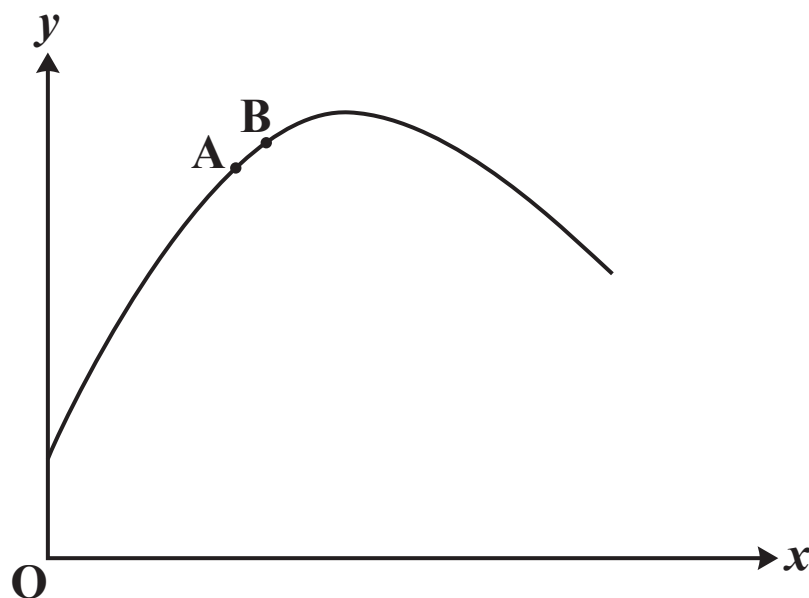
SECTION A (60 marks)

- 1 (a) Express $x^2 + 8x + 2$ in the form $(x + a)^2 + b$. [2]**
- (b) Write down the coordinates of the turning point of the curve $y = x^2 + 8x + 2$. [1]**
- (c) State the transformation(s) which map(s) the curve $y = x^2$ onto the curve $y = x^2 + 8x + 2$. [2]**
- 2 Solve the equation $\sin 2x = 0.3$ for $0^\circ \leq x \leq 180^\circ$. Give your answer(s) correct to 1 decimal place. [2]**
- 3 (a) Determine, in terms of k , the coordinates of the point where the lines with the following equations intersect.**
- $$x + y = k$$
- $$2x - y = 1 \quad [3]$$
- (b) Determine, in terms of k , the coordinates of the points where the line $x + y = k$ crosses the curve $y = x^2 + k$. [4]**

- 4 The diagram shows points A and B on the curve**

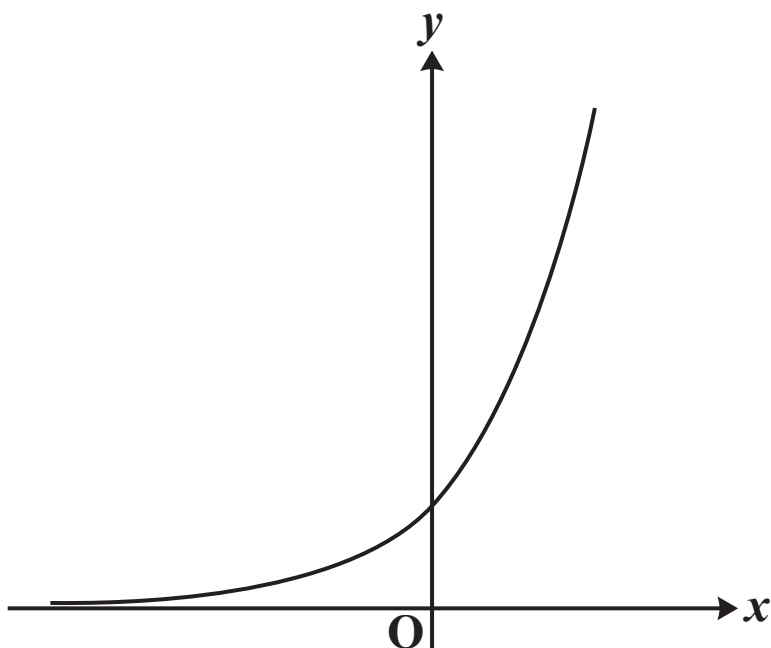
$$y = \left(\frac{x}{4}\right)^{-x}.$$

The x -coordinate of A is 1 and the x -coordinate of B is 1.1.



- (a) Find the gradient of chord AB. Give your answer correct to 2 decimal places. [2]**
- (b) Give the x -coordinate of a point C on the curve such that the gradient of chord AC is a better approximation to the gradient of the tangent to the curve at A. [1]**

- 5 (a) The diagram shows the curve $y = e^x$.



On the axes in the Printed Answer Booklet, sketch graphs of

(i) $\frac{dy}{dx}$ against x , [1]

(ii) $\frac{dy}{dx}$ against y . [2]

- (b) Wolves were introduced to Yellowstone National Park in 1995.

The population of wolves, y , is modelled by the equation

$$y = Ae^{kt},$$

where A and k are constants and t is the number of years after 1995.

(i) Give a reason why this model might be suitable for the population of wolves. [1]

(ii) When $t = 0$, $y = 21$ and when $t = 1$, $y = 51$.

Find values of A and k consistent with the data. [3]

(iii) Give a reason why the model will not be a good predictor of wolf populations many years after 1995. [1]

6 In this question you must show detailed reasoning.

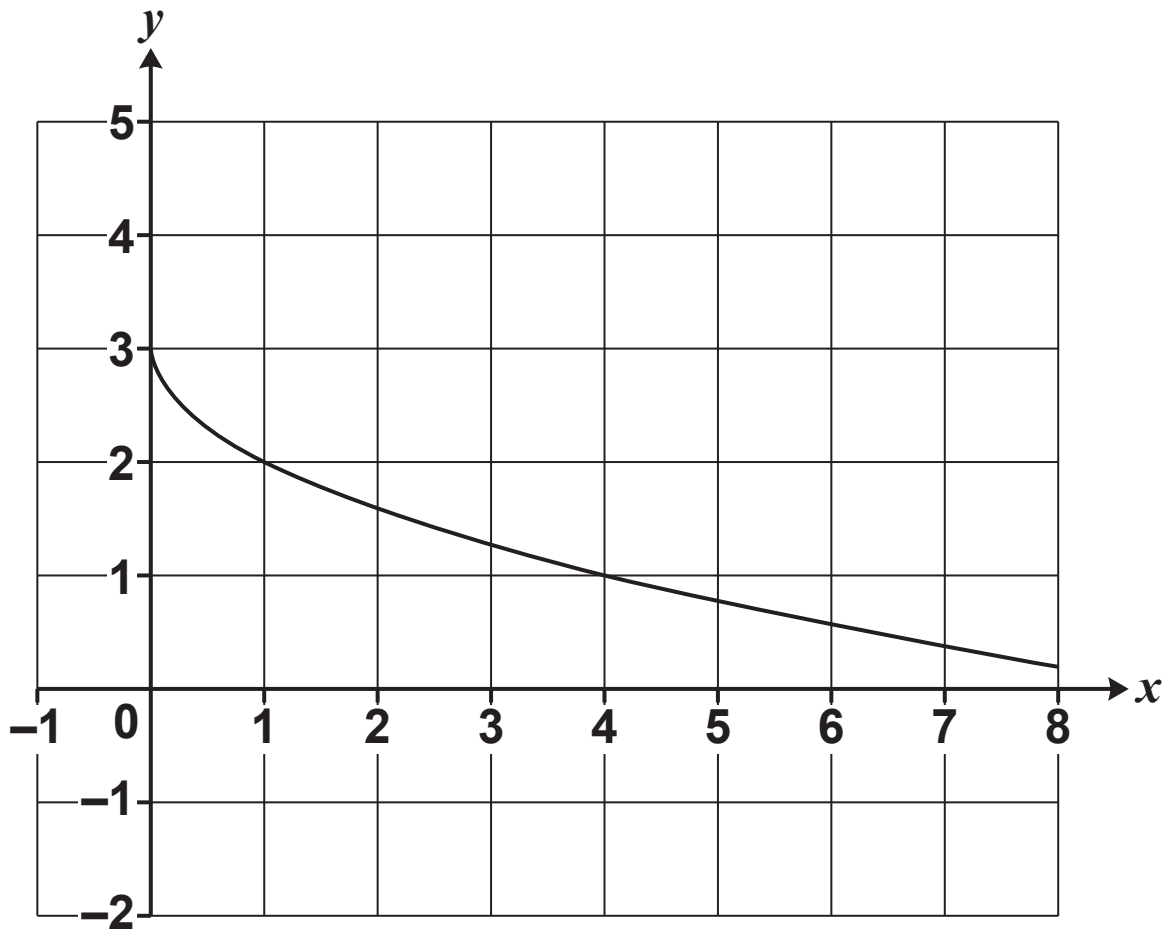
Show that $\sum_{r=1}^3 \frac{1}{\sqrt{r+1} + \sqrt{r}} = 1$. [4]

7 Determine $\int x \cos 2x \, dx$. [3]

8 For a particular value of a , the curve $y = \frac{a}{x^2}$ passes through the point (3, 1).

Find the coordinates of all the other points on the curve where both the x -coordinate and the y -coordinate are integers. [3]

9 The diagram shows the curve $y = 3 - \sqrt{x}$.



- (a) Draw the line $y = 5x - 1$ on the copy of the diagram in the Printed Answer Booklet. [1]
- (b) In this question you must show detailed reasoning.

Determine the exact area of the region bounded by the curve $y = 3 - \sqrt{x}$, the lines $y = 5x - 1$ and $x = 4$ and the x -axis. [10]

10 (a) Express $\frac{1}{(4x+1)(x+1)}$ in partial fractions. [3]

(b) A curve passes through the point (0, 2) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{y}{(4x+1)(x+1)},$$

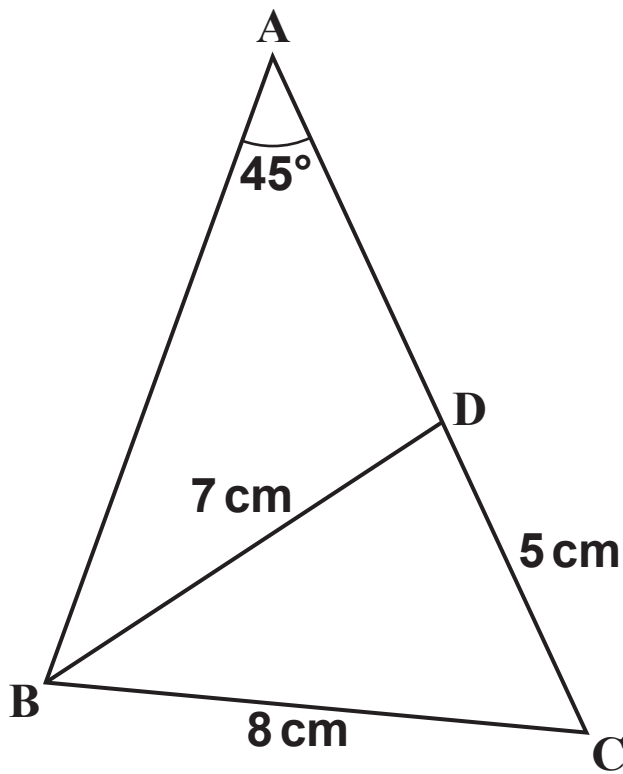
for $x > -\frac{1}{4}$.

Show by integration that $y = A\left(\frac{4x+1}{x+1}\right)^B$ where A and B are constants to be determined. [6]

11 In this question you must show detailed reasoning.

The diagram shows triangle ABC, with $BC = 8\text{ cm}$ and angle $BAC = 45^\circ$.

The point D on AC is such that $DC = 5\text{ cm}$ and $BD = 7\text{ cm}$.



Determine the exact length of AB. [5]

BLANK PAGE

Answer ALL the questions.

SECTION B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Show that $\beta = \arctan\left(\frac{1}{3}\right)$, as given in line 18. [3]

13 (a) Use triangle ABE in FIG. C2 to show that $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$, as given in line 40. [1]

(b) Sketch the graph of $y = \arctan x$. [1]

(c) What property of the arctan function ensures that $y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$, as given in line 41? [1]

14 (a) Show that

$$\begin{aligned} \arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) &= \arctan\left(\frac{1}{n}\right) \\ \Rightarrow \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) &= \arctan 1. \quad [1] \end{aligned}$$

(b) Use the arctan addition formula in line 30 to show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right), \text{ as}$$

given in line 58. [4]

15 Prove that $\arctan 1 + \arctan 2 + \arctan 3 = \pi$, as given in line 61. [4]

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.